**Influence of photosensor noise on accuracy of cost-effective Shack-Hartmann wavefront sensors**

**MOTIVATION AND GOALS**

Using comprehensive model of a photosensor, new results in numerical simulations and analysis of centroiding accuracy for the cost-effective CMOS-based wavefront sensors were elaborated:
- analysis of influence of different noise sources from the CMOS photosensor on the centroiding robustness in the Shack-Hartmann wavefront sensor is presented;
- influence of light and dark noises as well as pixelisation factor has been assessed.

Numerical experiments are aimed in study of a cost-effective wavefront sensor, and for that reason, we simulate CMOS sensors using previously developed high-level model.

**MODEL OF THE SHACK-HARTMANN WAVEFRONT SENSOR**

The Shack-Hartmann wavefront sensor was simulated with the parameters:
- $32 \times 32$ lenslets, $32 \times 32$ pixels per lenslet;
- photosensor in the WFS has $5.00 \mu m$ pixels with the pixel fill factor of 50%, quantum efficiency of 60%, and full well of 20000 $e^-$;
- surrounding temperature is 300K, the sense node gain is $5.0 \times V/e^- $, and clock speed is 20 MHz.

Such parameters are typical for a cost-effective CMOS-based wavefront sensor. The von Karman power spectrum model for the turbulence was used. The size of turbulence layer is $1024 \times 1024$ pixels.

**NUMERICAL SIMULATION RESULTS: LENSLET FOCAL LENGTH AND PIXELISATION ERROR INFLUENCE ON WAVEFRONT SENSOR ACCURACY**

**Lenslet focal length**

Conditions of the numerical experiment:
- observation wavelength $\lambda = 0.5 \mu m$;
- diameter of the lenslet $D_{\text{lenslet}} = 200 \mu m$;
- typical values for the focal distance of the lenslets in the Shack-Hartmann wavefront sensor are 10-20 mm;
- the focus distance of the lenslet from 1 mm to 50 mm with the step of 1 mm.

**Results:** The centroid position can be effectively measured using the lenslets with focal distance no more than 20-30 mm. Increasing the focus length further gives no improvements over the accuracy of centroiding.

**Pixelisation error**

Conditions of the numerical experiment:
- the amount of pixels in each lenslet was varied from $64 \times 64$ (initial size) down to $4 \times 4$ pixels with step of 4 pixels;
- size of the lenslet remains constant;
- the centroiding coordinates were calculated using a simple CoG algorithm both for the original centroiding image.

**Results:** The pixelisation error grows slowly when the number of pixels in the lenslets shrinks from $64 \times 64$ to $32 \times 32$ pixels. The pixelisation error in this case is $\Delta x_{\text{FSR}} = 0.002$ or 0.2% rms.

Further increase of the PRNU factor to 5% shows the increase of the pixelisation error to $0.3\ldots0.6 \%$ in low-level area of measurements up to $0.4\ldots0.9 \%$.

**NUMERICAL SIMULATION RESULTS: CENTROIDING ERROR CAUSED BY THE PHOTON SHOT NOISE**

**Photon shot noise influence on accuracy**

Conditions of the numerical experiment:
- $32$ images of the centroids were generated given the same turbulence realization;
- the standard deviation of the centroids coordinates was calculated in the number of pixels;
- results of the photon shot noise influence are presented as the percentage of the centroid’s coordinate.

**Results:** As the integration time increases, the shot noise increases proportionally. However, in different lenslets, centroids are different; hence, the dependency may look different. The analysis of the data allows to say that the influence of shot noise is from 0.2% to 1.5% of pixel.

**Probability density of centroiding errors due to shot noise**

The growth of the pixelisation error increases: for the case of $12 \times 12$ pixels in the lenslet one can observe an abrupt increase of the pixelisation error to $\Delta x_{\text{FSR}} = 0.007$, which is 3 times larger than for the case $32 \times 32$ pixels in the lenslet.

**NUMERICAL SIMULATION RESULTS: CENTROIDING ERROR CAUSED BY THE PRNU NOISE**

**PRNU noise influence on accuracy**

Conditions of the numerical experiment:
- $32$ images were of the centroids generated with the same turbulence realization;
- the standard deviation of the centroids coordinates was calculated in the number of pixels;
- the PRNU factor was 1%, 2% and 5%.

**Results:** The increase of the PRNU factor from 1% to 2% also increases the centroiding error, especially in low-light area (below 3000 electrons). The increase of the centroiding error from $0.3\ldots0.6 \%$ in low-level area of measurements up to $0.4\ldots0.9 \%$.

Further increase of the PRNU factor to 5% shows the increase of the centroiding errors to $0.9\ldots1.7 \%$, which makes the wavefront estimation difficult.

The exact behaviour of errors in different lenslets may differ, but the overall influence is similar.

**Probability density of centroiding errors due to PRNU noise**

The table shows the quantisation errors produced by 10-bit ADCs introduce mean error in centroids coordinates of 22% in low-light conditions. The ADC with 12 bit resolution should be used in cost-effective WFS instead of commonly used 10-bit ADCs.

**Table:**

<table>
<thead>
<tr>
<th>Light conditions</th>
<th>Double precision (no quantisation)</th>
<th>12 bit quantisation</th>
<th>10 bit quantisation</th>
<th>8 bit quantisation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>max</td>
<td>mean</td>
<td>max</td>
</tr>
<tr>
<td>Low light level</td>
<td>5.2%</td>
<td>6.1%</td>
<td>6.4%</td>
<td>12.8%</td>
</tr>
<tr>
<td>Medium light level</td>
<td>3.9%</td>
<td>4.3%</td>
<td>4.9%</td>
<td>5.2%</td>
</tr>
<tr>
<td>High level of light</td>
<td>3.3%</td>
<td>4.1%</td>
<td>3.9%</td>
<td>4.2%</td>
</tr>
</tbody>
</table>

**Results:**

The quantisation errors produced by 10-bit ADCs introduce mean error in centroids coordinates of 22% in low-light conditions. The ADC with 12 bit resolution should be used in cost-effective WFS instead of commonly used 10-bit ADCs.

**Conclusions**

1. assuming that WFS has $64 \times 64$ pixels per lenslet, the pixelisation error is shown to grow slowly until the amount of pixels in lenslets are half of original (causes the error of 0.2% rms);
2. the pixelisation error increases abruptly to 0.7% rms (3 times increase) when the number of pixels in the lenslet is a quarter of the original amount.
3. the centroiding errors caused by photon shot noise only and for the case of shot noise and PRNU is of the same order (up to 1.5% of pixel).
4. the PRNU influence on the centroiding accuracy for the PRNU factor of 5%, 2% and 1% is $0.9\ldots1.7 \%$ of pixel, $0.4\ldots0.9 \%$ of pixel and $0.3\ldots0.6 \%$ of pixel, respectively.
5. the results for quantisation errors show that the 10-bit ADCs, which are frequently used in cost-effective WFS, may be considered as a sub-optimal solution.
6. in low-light conditions, the mean error in centroids coordinates for 10 bit ADC is of 22%, whenever for the 12-bit ADC the mean error is only 6%.
7. for the photon shot noise and PRNU, the distribution of centroids errors tends to drift from the Gaussian-like to the non-symmetrical distribution.

**Mikhail V. Konnik**

**James Welsh**

School of Electrical Engineering and Computer Science
The University of Newcastle, Australia